

# Countercurrent Stagewise Operations

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10. Design of a Tray Benzene Absorption Tower. A countercurrent tray absorption column is fed at the bottom with an air stream containing 1.5 mole% benzene and the balance air. It is desired to recover 99% of the benzene in the feed by countercurrent contact with a non-volatile oil which enters at the top of the column containing 0.2 mole% benzene. The column operates at 27°C. At this temperature the vapor pressure of benzene is 103.6 mmHg. Draw a schematic showing all the problem data on the schematic and design the column using a reasonable pressure and solvent rate. Report the required number of real trays for an over-all column efficiency of 75% and the column height for a tray spacing of 18 inches. Caution: as the inlet solvent is not pure, if the column pressure is too low the equilibrium and operating lines may cross. State all assumptions. ....	46

A common method of carrying out phase separations is to contact the phases in discrete stages with the phases flowing countercurrent to each other, that is, in opposite directions, through the stages. The complete set of contact stages is known as a *cascade*. A schematic of a countercurrent cascade is shown in Fig. 1.

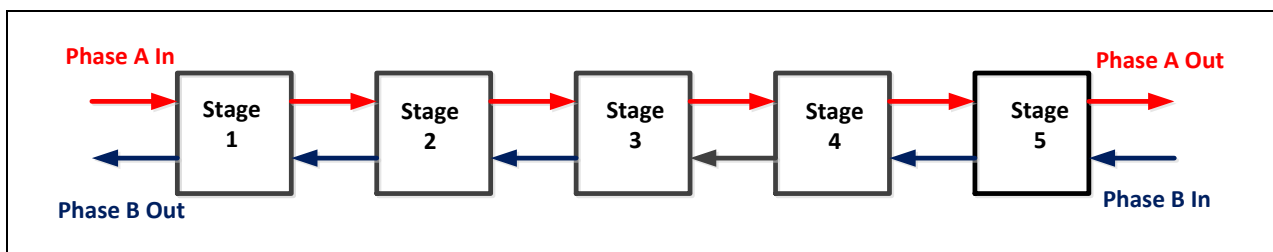


Figure 1. Countercurrent Cascade

In separations involving solid slurries such as leaching, or viscous liquid phases such as liquid-liquid extraction, each of the stages shown in Fig. 1 may

consist of at least two pieces of equipment, a digester to contact the phases and a decanter to separate them, as shown in Fig. 2.

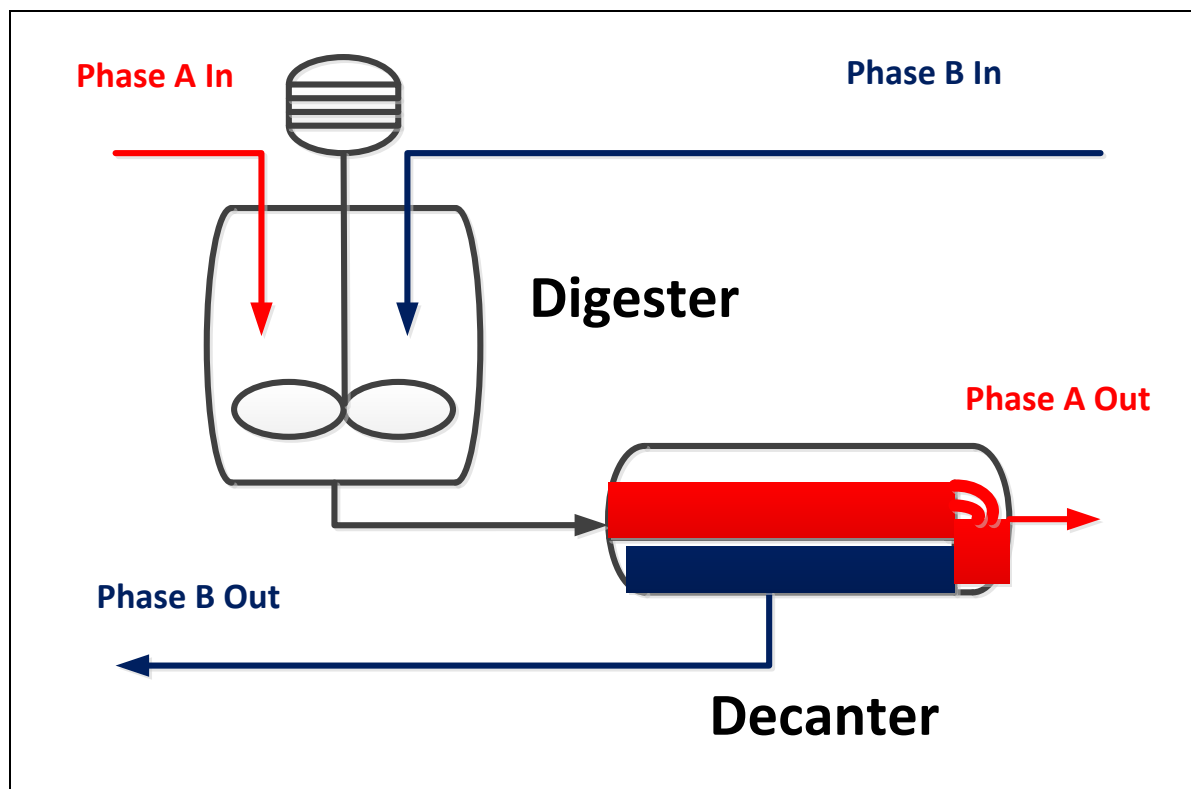


Figure 2. Separation Stage Consisting of a Digester and a Decanter

Some operations such as absorption, stripping, distillation and liquid-liquid extraction are carried out in tray towers where each tray or plate is a separation stage, as shown in Fig. 3. The liquid phase cascades down from tray to tray under the action of gravity while the vapor flows up, bubbling through the liquid on the trays and driven by a pressure differential. In liquid-liquid extraction the denser of the two liquid phases enters at the top of the tower and flows down by gravity while the less dense phase enters at the bottom and flows up by buoyancy.

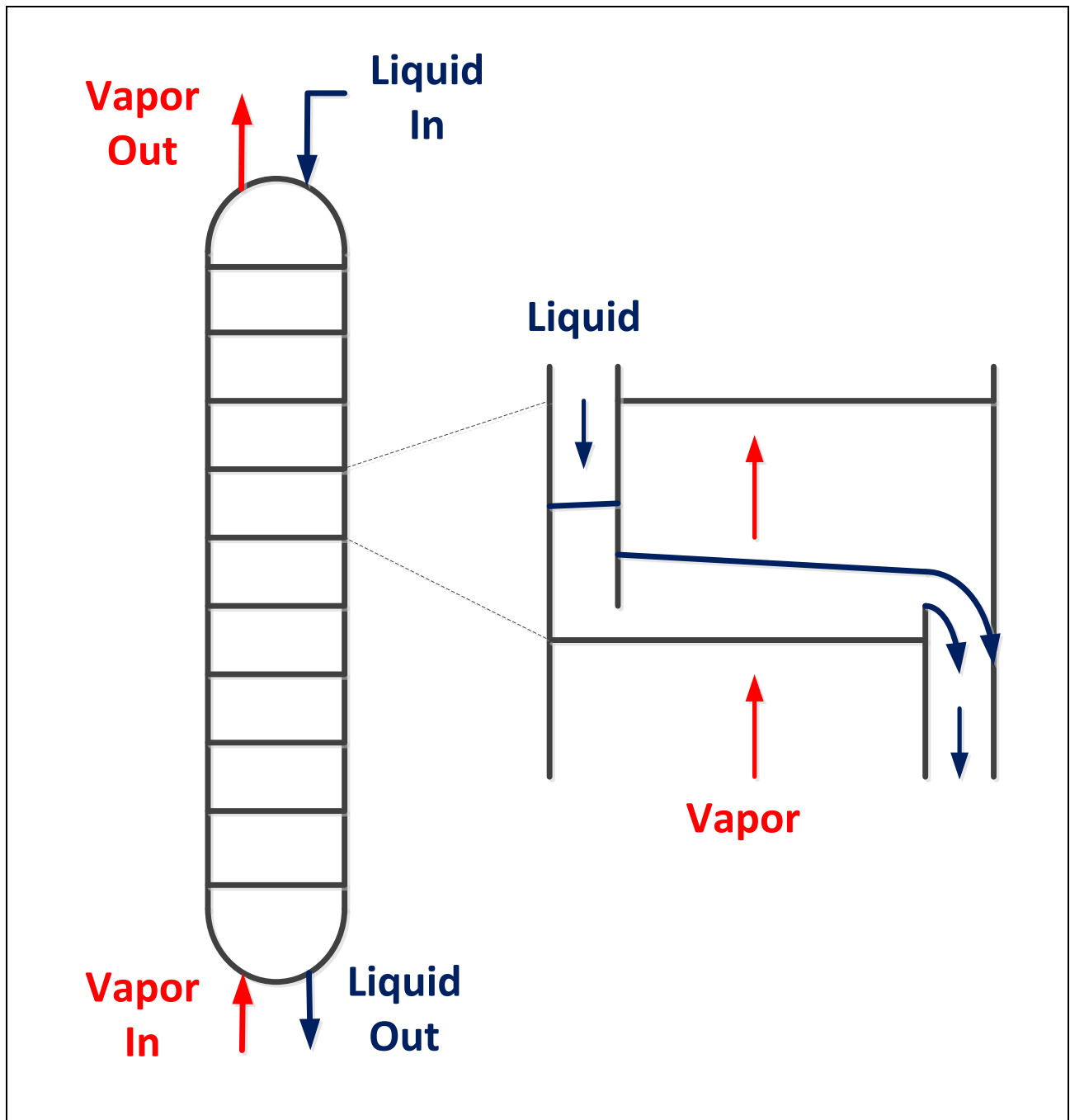


Figure 3. Tray Tower and Typical Separation Tray

### ***1 Equilibrium Stage***

In each separation stage the two phases in contact approach their equilibrium compositions, but they cannot reach equilibrium because there is not enough

contact time and other non-idealities. However, the common design procedure for stage separations is to assume the two phases reach equilibrium. In other words, each stage in the cascade is assumed to be an equilibrium stage for design purposes. This is why this design topic is called *Countercurrent Equilibrium Stage Separations*. The following is the definition of an *equilibrium stage*:

*An equilibrium stage is one in which the compositions of the streams leaving the stage are in equilibrium with each other.*

Two important things to remember about this definition are:

- It is the streams *leaving* the stage that are in equilibrium with each other
- The streams leaving the *cascade* of stages are not in equilibrium with each other.

You may have heard it said that once two phases reach equilibrium there is no longer any net mass transfer between them and you may ask, how can there be mass transfer in an equilibrium stage? The answer is that the phases entering an equilibrium stage are not in equilibrium with each other so that, as they approach equilibrium some of the solute is transferred from one phase to the other.

### 1.1 Number of Actual Stages.

Because of the equilibrium assumption the design procedure determines the number of equilibrium stages required to achieve a specified separation. Equilibrium is not normally achieved in a stage because there is not enough contact time or because of imperfect mixing and other non-idealities. To determine the actual number of stages the simplest procedure is to assume an *over-all efficiency* for the cascade or column and divide the number of equilibrium stages by the over-all efficiency to estimate the number of actual stages:

$$N_a = \frac{N}{\eta} \quad (1)$$

Where  $N_a$  is the number of actual stages or trays,  $N$  is the number of equilibrium stages or trays, and  $\eta$  is the over-all efficiency.

The over-all efficiency is sometimes estimated from experience with the type of separation equipment being designed, determined experimentally, or estimated from mass transfer coefficients.

## **2 Definition of The Design Problem**

The design problem for a cascade of stages is defined as: *given the feed flow and composition and the desired product composition or recovery of the component of interest, determine the required number of stages.*

As in any design problem the designer must be allowed to select a *design variable* to complete the definition of the problem. This means that a design problem is not completely defined. The following are the typical design variables for several separation operations:

- The solvent inlet rate for absorption, leaching and liquid-liquid extraction or, equivalently, the composition of the final extract.
- The vapor inlet rate in stripping or, equivalently, the composition of the outlet vapor stream.
- The reflux ratio in a distillation column.
- The fraction vaporized in a flash distillation drum.

An additional required specification in absorption, leaching and extraction is the inlet composition of the solvent and in stripping the inlet vapor composition. When these are not specified they can be assumed to be zero, that is, pure inlet solvent or vapor stream. Also, as the number of stages is not a function of the total feed rate, when the feed rate is not specified a basis can be assumed for it.

The complete design of the separation operation requires also the size of the equipment, that is, the diameter of the column or the sizes of the digesters and decanters. These depend on the total feed flow and their estimation is specific to each type of operation.

Solution of the design problem is carried out in two steps, the calculation of the outlet flows and compositions from over-all material balances on the entire cascade followed by the calculation of the number of stages. The next two sections address each of these steps.

### ***3 Calculation of Inlet and Outlet Flows and Compositions***

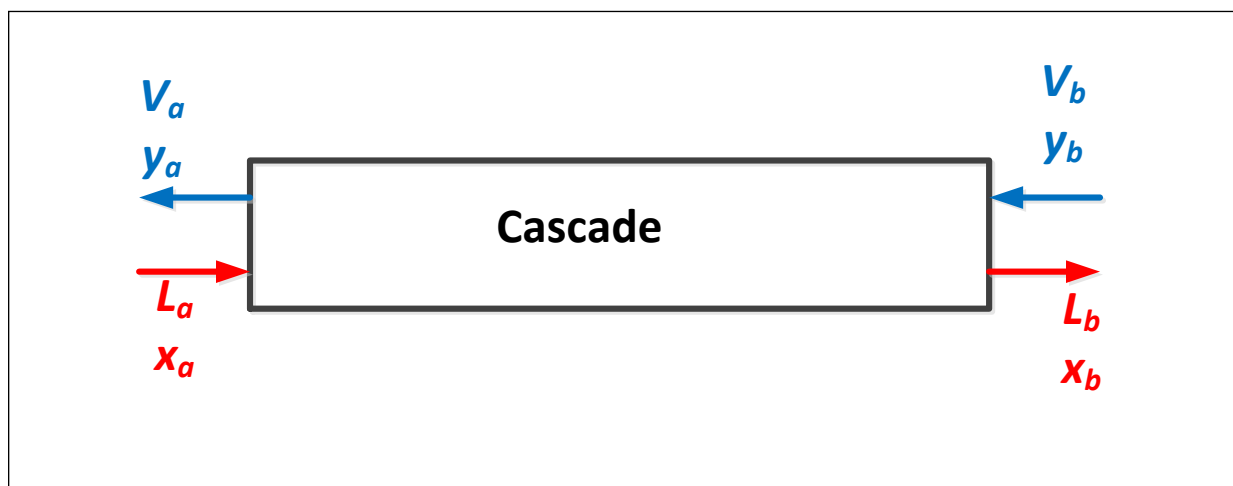
The inlet flows and compositions of the entire cascade are calculated from over-all material balances on the total flow and each of the components entering and exiting the operation. To do this you must recall a fundamental principle of material balances:

*On any system the number of independent material balances is equal to the number of components.*

This means that although we can write a balance on total mass and a balance on the mass of each component, one of those balances is not independent of the others. When modeling a process system it is essential to be able to

determine how many independent relationships can be established between the variables of the system so as to know if the problem can be solved. This is one important skill you must acquire if you are to become an engineer who can formulate the solution to new problems, an exciting prospect, or just one who only knows how to plug numbers into formulas that others have developed, a boring and tedious task.

In writing material balances it is important to draw a schematic of the system over which the balances are written, showing all the streams as arrows in or out of the system and labeling each stream with the variables associated with it, usually flow and composition. For the over-all balances on a cascade the system can be represented by a simple box with the inlet and outlet



streams, as in Fig. 4.

Figure 4. Schematic for Over-All Material Balances on a Cascade

The notation we will use here is as follows:

$L$  = the flow of the liquid or heavier stream

$V$  = the flow of the vapor or lighter stream

$x$  = composition of the liquid or heavier stream



$y$  = composition of the vapor or lighter stream

subscript  $a$  = point of entrance of the  $L$  stream and exit of the  $V$  stream

subscript  $b$  = point of exit of the  $L$  stream and entrance of the  $V$  stream

Regarding the units of these variables they are selected to facilitate the solution of the equations and this depends on the type of separation and other conditions of the problem. Nevertheless, they must be consistent. For example, if the flows  $L$  and  $V$  are in molar flows, the compositions  $x$  and  $y$  must be mole fractions, if the flows are weight flows the compositions must be weight fractions, etc.

The over-all material balances can be a total mass balance and a component mass balance for all the components except one, or a component mass balance for each component without a total mass balance. For design of continuous separation processes steady state is assumed, so the balances are simply the sum of the flows in is equal to the sum of the flows out.

The total mass balance is:

$$L_a + V_b = L_b + V_a \quad (2)$$

The component mass balance is:

$$L_a x_a + V_b y_b = L_b x_b + V_a y_a \quad (3)$$

When there are more than two components we can write one component balance for each component or a total balance and component balances for

each component except one. Recall that the number of independent material balances we can write is equal to the number of components.

With the over-all material balances the flows and compositions of all the streams in and out of the cascade that are not specified or assumed are calculated. With these it is then possible to estimate the number of stages required to do the specified separation, which is the topic of the next section.

#### ***4 Determination of the Required Number of Stages***

The number of equilibrium stages required to achieve the specified separation is determined from two relationships:

- The equilibrium relationship between the compositions leaving each stage
- Internal material balances relating the compositions in and out of each stage.

The calculation can be performed stage by stage, starting at either end of the cascade and ending when the composition leaving a stage reaches the calculated composition at the other end. As this method can be rather tedious and time consuming when many stages are required, a graphical procedure has been developed to carry it out, but to better understand the graphical procedure we will first discuss the stage by stage procedure.

Consider the cascade sketched in Fig. 5 keeping in mind that the compositions and flows in and out of the system have been determined from the over-all balances.

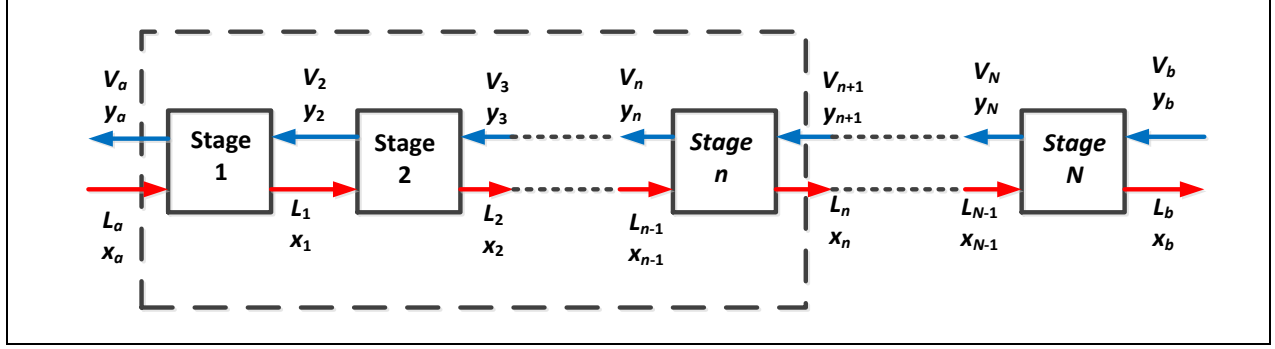


Figure 5. Schematic of Cascade Showing the Individual Stages

Notice that in the notation used here the subscript of each variable is the number of the stage from which the stream exits.

By the definition of an equilibrium stage, the equilibrium relation gives us the compositions leaving each stage, that is,  $x_1$  as a function of  $y_a$ ,  $x_2$  as a function of  $y_2$ , and so on. Also, experimental data on the separation equipment (tray, decanter, etc.) give us the relation between the  $L$  flow as a function of the composition  $x$ , that is,  $L_1$  as a function of  $x_1$  and so on.

To complete the relations required to solve the stage by stage calculations, we write material balances around the first  $n$  stages of the cascade, that is, around the dashed box shown in Fig. 5. From the total mass balance:

$$V_{n+1} = V_a + L_n - L_a \quad (4)$$

From the component mass balance:

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{V_a y_a - L_a x_a}{V_{n+1}} \quad (5)$$

The stage by stage calculation is then carried out as follows:

1. Determine  $x_1$  from  $y_a$  using the equilibrium relationship.
2. Determine  $L_1$  from  $x_1$  using the experimental data from the equipment.
3. Calculate  $V_2$  for  $n = 1$  in Eq. (4).
4. Calculate  $y_2$  for  $n = 1$  in Eq. (5).
5. Determine  $x_2$  from  $y_2$  using the equilibrium relationship.
6. Repeat steps 2 thru 5 for stage 2 ( $n = 2$ ), then stage 3, and so on.
7. Stop when  $x_N$  reaches the value of  $x_b$ . Then  $N$  is the required number of stages.

Seldom will the composition of the last stage exactly match the value of  $x_b$ , so the last stage may be calculated as a fraction of a stage. You may ask, how can one have a fraction of a stage? The answer is that, as we are calculating the number of *equilibrium stages*, we can have a fraction of equilibrium stage that, when corrected for actual stages, will be rounded up to an integer number of actual stages.

The above procedure is very amenable for programming on a workbook or spreadsheet if the equilibrium and flow relationships can be expressed as formulas. Carrying it out manually can be tedious when a large number of stages is required. An alternative is to carry out the procedure graphically which is also facilitated when the equilibrium relationship is in the form of a graph.

## **5 Graphical Determination of the Number of Stages—The McCabe-Thiele Method**

To facilitate the procedure for determining the required number of stages, two graduate students from MIT, McCabe and Thiele<sup>1</sup>, developed a graphical

procedure that is named after them. It is carried out on the  $x$ - $y$  diagram which plots the composition  $y$  of the vapor or light phase versus the composition  $x$  of the liquid or heavy phase. Two lines are plotted, one relating the equilibrium compositions leaving each stage is called the *equilibrium line*, and the other one plotting the composition of the streams in and out of each stage, a plot of Eq. (5), is called the *operating line*. These lines are shown in the  $x$ - $y$  diagram of Fig. 6. The equilibrium line is obtained from experimental data and the operating line is obtained from numbers calculated with Eq. (5). Notice that the two end points of the operating line are  $(x_a, y_a)$  and  $(x_b, y_b)$  which are determined from the over-all mass balances. Usually one other point in the middle of this range can be calculated from Eq. (5) to sketch the line when it is curved.

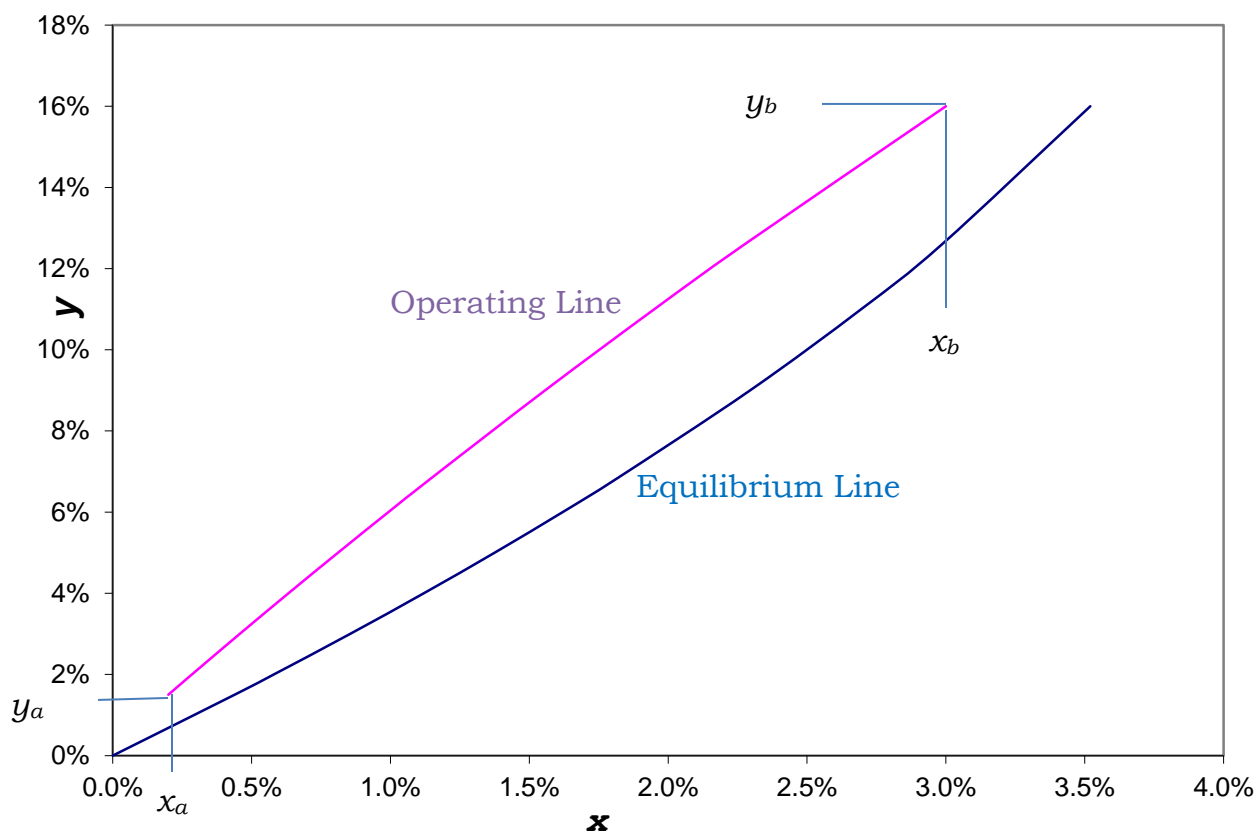


Figure 6 . McCabe-Thiele  $x$ - $y$  Diagram

Once the operating and equilibrium lines plotted in the  $x$ - $y$  diagram, we are ready to step the stages. The procedure is as outlined in the previous section, but the compositions are determined graphically. Figure 7 shows the stages.

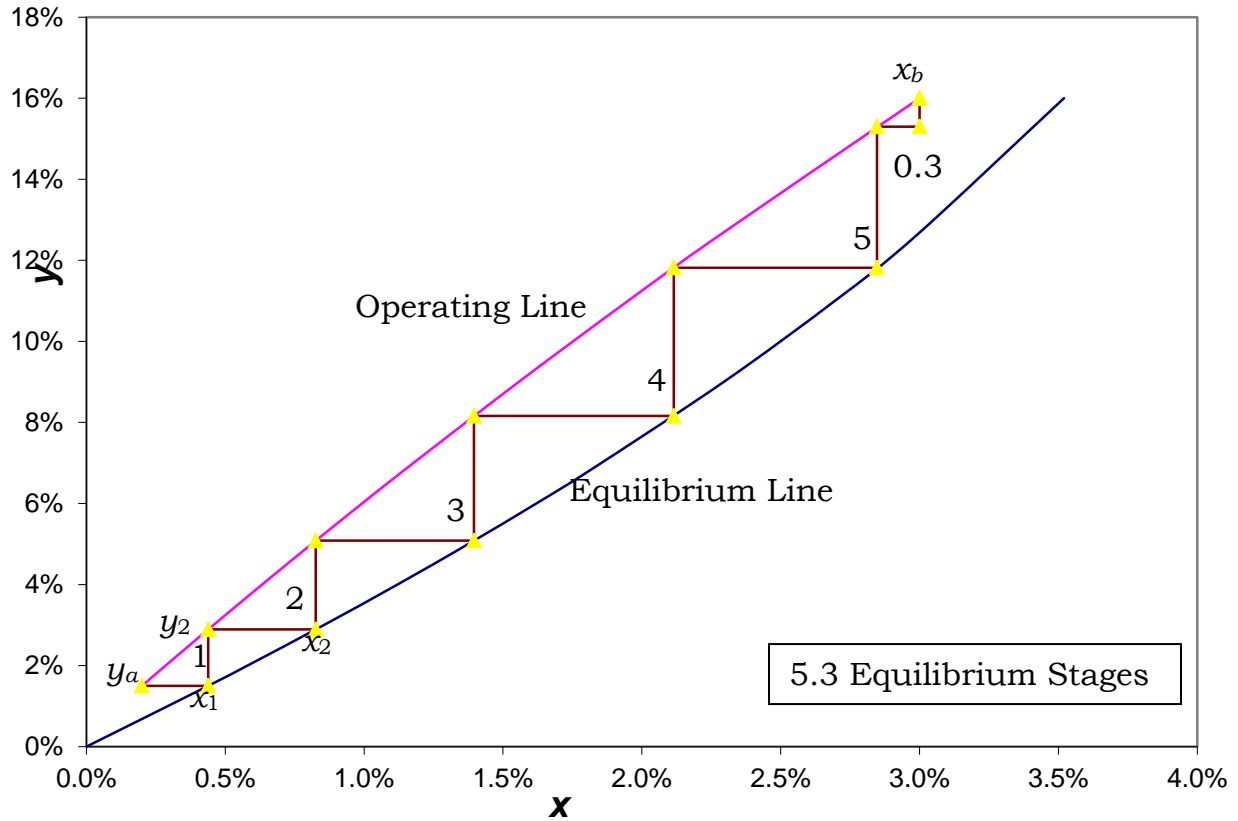


Figure 7. McCabe-Thiele Determination of the Number of Required Stages

The procedure is as follows:

1. Starting at  $y_a$  we move to the right to the equilibrium line to find  $x_1$  as  $x_1$  and  $y_a$  leave stage 1 and are in equilibrium (see Fig. 5).
2. Then we move up to the operating line to find  $y_2$  as the operating line is a plot of Eq. (5) with  $n = 1$ .
3. We then move to the right to the equilibrium line to find  $x_2$  as  $x_2$  and  $y_2$  leave stage 2 and are in equilibrium.
4. We continue stepping from one line to the other until the value of  $x$  is greater than  $x_b$ , in this case after stage 5. We see that less than one stage

is required to reach that value, in this case 0.3 of a stage. So we have determined that 5.3 equilibrium stages are required.

The McCabe-Thiele procedure can be applied to any countercurrent stage separation operation. In the example of Figs. 6 and 7 the operating line falls above the equilibrium line. This is characteristic of the absorption operation in which mass transfer is from the vapor to the liquid. In most other operations (stripping, distillation, leaching, extraction, etc.) the operating line falls below the equilibrium line because mass transfer is from the liquid to the vapor.

Why is this graphical procedure important to learn in this age in which computer programs are available to precisely carry out the calculations required to solve the design problems? Because graphs help us understand how our decisions affect the resulting design. For example, graphically we can see how our selection of the solvent rate affects the relative positions of the operating and equilibrium lines, the number of required stages, and the composition of the final extract. Without this understanding we can arrive at poor designs when we use the computer programs and not be able to see how to improve them.

Well rounded engineers use schematics and graphs to better understand the scientific principles involved in our work. The right brain, which is where our creativity resides, is the one that understands graphs. You should take advantage of every opportunity to exercise your right brain.

## ***6 The Kremser Equation***

When both the equilibrium and operating lines are straight, or approximately straight, an equation can be derived to estimate the required number of equilibrium stages; it is called the Kremser equation. It must not be confused

with the equation that is used to estimate the number of transfer units in a packed tower when the equilibrium and operating lines are straight. Here we will present the equation and then the derivation. The derivation is presented to show that the Kremser equation is an exact equation, not an experimental correlation, when the equilibrium and operating lines are straight. Familiarizing yourself with the correlation is important to understand the restrictions to apply to the equation. It also helps you practice your algebra skills which may be getting rusty.

### 6.1 Kremser Equation for Absorption.

In absorption operations mass is transferred from the vapor phase to the liquid phase. This requires that the composition of the vapor phase, plotted by the operating line, be higher than the composition of the vapor in equilibrium with the liquid phase, plotted by the equilibrium line, as in Fig. 8.

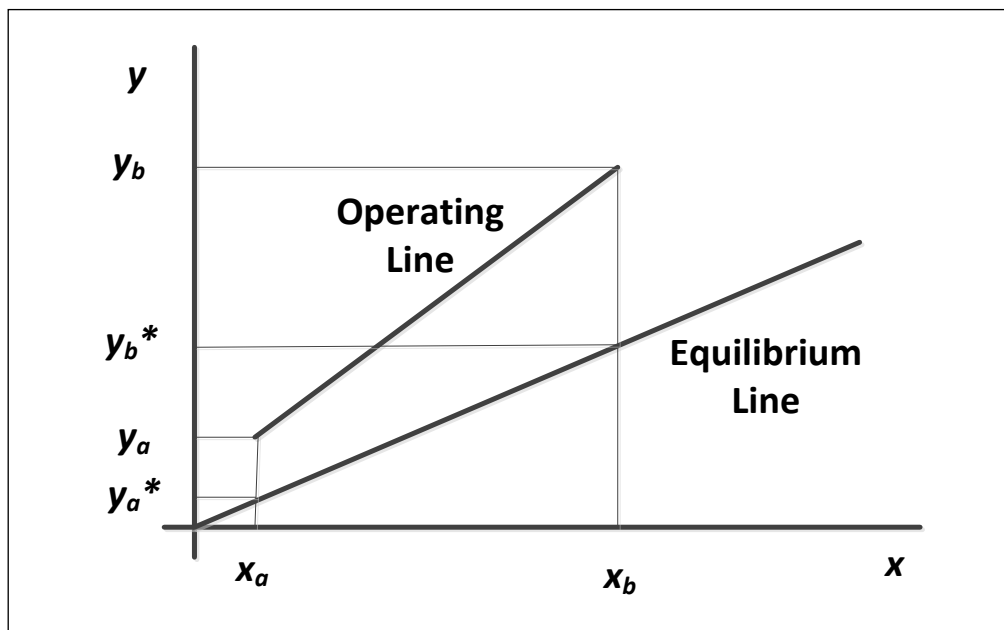


Figure 8.  $x$ - $y$  Diagram for Absorption Operation



The Kremser equation for this operation is:

$$N = \frac{\ln\left(\frac{y_b - y_b^*}{y_a - y_a^*}\right)}{\ln\left(\frac{y_b - y_a}{y_b^* - y_a^*}\right)} \quad (6)$$

Where  $N$  = number of required equilibrium stages

$x_a, y_a$  = composition of liquid and vapor phases at liquid inlet

$x_b, y_b$  = composition of liquid and vapor phases at liquid exit

$y_a^*$  = composition of vapor in equilibrium with liquid at  $x_a$

$y_b^*$  = composition of vapor in equilibrium with liquid at  $x_b$

Notice that the fraction in the numerator of the formula is the ratio of the mass transfer driving force at the liquid exit side to the driving force at the liquid inlet side, and the denominator is the ratio of the slope of the operating line to the slope of the equilibrium line. The formula results in division by zero if the two lines are parallel (as the  $\ln(1.0) = 0$ ). So when the operating and equilibrium lines are parallel, the formula becomes:

$$N = \frac{y_b - y_a}{y_b - y_b^*} \quad (7)$$

When the two lines are parallel is the only instance that the number of equilibrium stages is the same as the number of transfer units in a packed tower.

As an example of the use of the Kremser equation let us apply it to the case of Fig. 7. From the figure we can read the following:

$$y_b = 0.16 \qquad y_b^* = 0.13 \text{ (at } x_b = 0.030\text{)}$$

$$y_a = 0.015 \qquad y_a^* = 0.007 \text{ (at } x_a = 0.002\text{)}$$

$$N = \frac{\ln\left(\frac{0.16 - 0.13}{0.015 - 0.007}\right)}{\ln\left(\frac{0.16 - 0.015}{0.13 - 0.007}\right)} = 8$$

As compared with 5.3 stages by the McCabe-Thiele graphical procedure of Fig. 7. The reason the Kremser equation results in more stages than the graphical procedure is that in this case the equilibrium and operating lines curve away from each other. As the Kremser equation is based on the lines being straight between the two end points, the lines would be closer to each other and more stages would be required as each stage would result in a smaller change in composition. You can check this out with a ruler on Fig. 7.

## 6.2 Kremser Equation for Stripping Operations.

When mass transfer is from the liquid the vapor phase, as in stripping, distillation, leaching, extraction and others, the composition of the liquid, plotted by the operating line, is higher than the composition of the liquid in equilibrium with the vapor, plotted by the equilibrium line, as in Fig. 9.

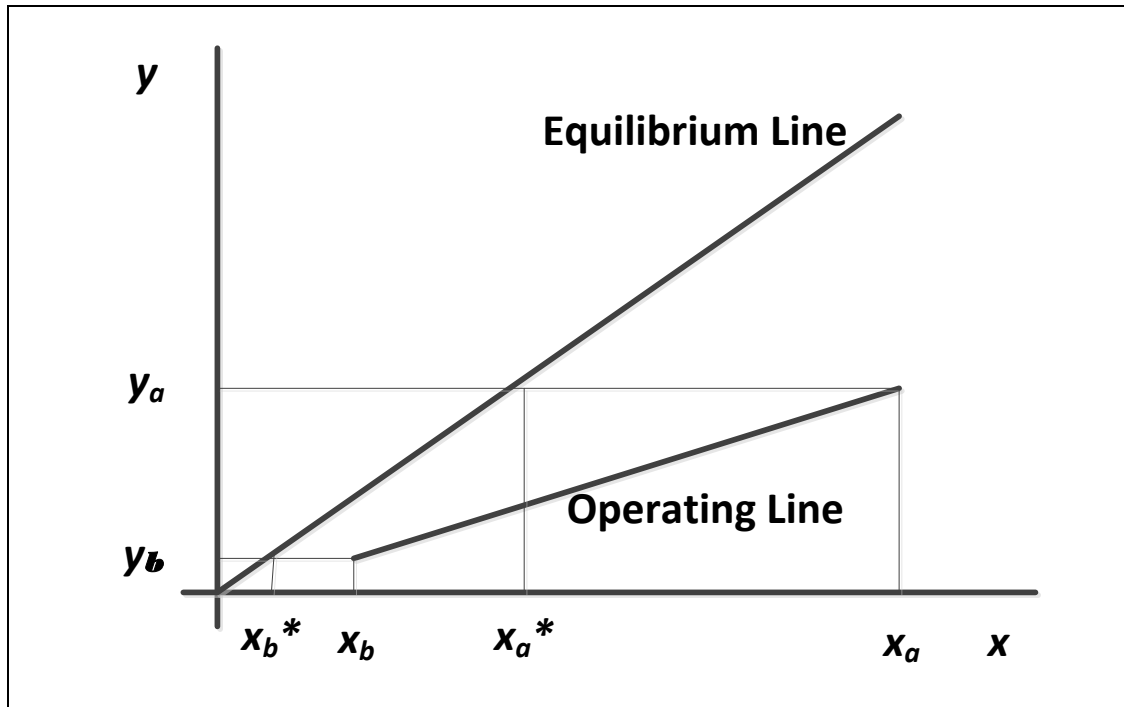


Figure 9.  $x$ - $y$  Diagram for Stripping Operations

The Kremser formula for stripping operations is:

$$N = \frac{\ln\left(\frac{x_a - x_{a*}}{x_b - x_{b*}}\right)}{\ln\left(\frac{x_a - x_b}{x_{a*} - x_{b*}}\right)} \quad (8)$$

When the operating and equilibrium lines are parallel, the formula becomes:

$$N = \frac{x_a - x_b}{x_a - x_{a*}} \quad (9)$$

When the operating and equilibrium lines are exactly straight, both the absorption and stripping formulas give exactly the same result, but when the lines are only approximately straight and the formulas are used to get an approximate estimate of the number of equilibrium stages, it is important to use the correct formula for absorption or stripping, whichever is the case.

### 6.3 Derivation of the Kremser Equation

Let the straight equilibrium line relating the compositions leaving each stage,  $y_n$  and  $x_n$  be represented by the equation:

$$A \quad y_n = B + mx_n$$

Where  $m$  is the slope of the equilibrium line and  $B$  is the intercept.

For the operating line to be straight the liquid and vapor rates in Eq. (5) must be constant, that is  $L_n = L$  and  $V_{n+1} = V$ . So, Eq. (5) becomes:

$$B \quad y_{n+1} = \frac{L}{V}x_n + y_a - \frac{Lx_a}{V}$$

Combine A and B to eliminate  $x_n$ :

$$C \quad y_{n+1} = \frac{L}{mV}(y_n - B) + y_a - \frac{L}{mV}mx_a$$

Substitute  $A = \frac{L}{mV}$ :

$$D \quad y_{n+1} = Ay_n + y_a - A(mx_a + B)$$

From the equilibrium line, let  $y_a^* = B + mx_a$ , and substitute

$$E \quad y_{n+1} = Ay_n + y_a - Ay_a^*$$

For  $n = 1$ ,  $y_1 = y_a$  (the liquid composition leaving stage 1, see Fig. 5), substitute:

$$F \quad y_2 = Ay_a + y_a - Ay_a^* = (1 + A)y_a - Ay_a^*$$

Substitute  $n = 2$  in E and then substitute  $y_2$  from F in E:

$$G \quad y_3 = Ay_2 + y_a - Ay_a^* = A(1 + A)y_a + y_a - A^2y_a^* - Ay_a^* \\ = (1 + A + A^2)y_a - (A + A^2)y_a^*$$

We continue stage by stage until  $n = N$ , and notice that  $y_{N+1} = y_b$  (the vapor composition entering stage  $N$ , see Fig. 5):

$$H \quad y_{N+1} = y_b = (1 + A + A^2 + \dots + A^N)y_a - (A + A^2 + \dots + A^N)y_a^*$$

The terms in parenthesis in Eq. H can be summed as follows:

$$I \quad y_b = \frac{1-A^{N+1}}{1-A}y_a - \frac{A(1-A^N)}{1-A}y_a^*$$

$$\text{Simplify:} \quad y_b - Ay_b = y_a - A^{N+1}y_a - Ay_a^* + A^{N+1}y_a^*$$

$$J \quad A^{N+1}(y_a - y_a^*) = A(y_b - y_a^*) - (y_b - y_a)$$

From Eq. E for  $n = N$ :

$$K \quad y_{N+1} = Ay_N + y_a - Ay_a^*$$

Now,  $y_{N+1} = y_b$ , and, as  $y_N$  and  $x_b$  leave stage  $N$  (see Fig. 5),  $y_N = y_b^*$  (in equilibrium with  $x_b$ ); substitute into Eq. K and rearrange:

$$L \quad y_b - y_a = A(y_b^* - y_a^*)$$

Substitute L into J:

$$M \quad A^{N+1}(y_a - y_a^*) = A(y_b - y_a^*) - A(y_b^* - y_a^*)$$

Simplify and rearrange Eq. M:

$$A^N(y_a - y_a^*) = y_b - y_a^* - y_b^* + y_a^*$$

$$A^N = \frac{y_b - y_b^*}{y_a - y_a^*}$$

Take logarithms and solve for  $N$ :

$$N = \frac{\ln\left(\frac{y_b - y_b^*}{y_a - y_a^*}\right)}{\ln A}$$

Equation N is one of the forms of the Kremser equation that can be used with  $A = L/mV$  when the equilibrium line is straight and the liquid and vapor rates are constant. In many cases the Kremser equation can be used to obtain a rough estimate of the number of stages when the equilibrium and operating lines are only approximately straight. Then, as  $L$ ,  $V$  and  $m$  vary throughout the cascade, an average value of  $A$  is necessary. We get it by rearranging Eq. L:

$$A = \frac{y_b - y_a}{y_b^* - y_a^*}$$

Substitute Eq. O into Eq. N:

$$N = \frac{\ln\left(\frac{y_b - y_b^*}{y_a - y_a^*}\right)}{\ln\left(\frac{y_b - y_a}{y_b^* - y_a^*}\right)} \quad \text{Q.E.D.}$$

Equation P is Eq. (6) presented above for absorption operations. A similar derivation can be made of Eq. (8) for stripping operations.

## 6.4 The Absorption and Stripping Factors

As mentioned earlier, a design problem is an open-ended problem, that is, it is not completely defined. There is usually at least one design variable that must be selected to arrive at a reasonable design. By “reasonable” we mean that it does not require an excessive number of stages or produces a very dilute extract. As it is not possible to evaluate every possible value of the design variable, the engineer often uses factors that experience has shown result in reasonable design. Such factors are the absorption and stripping factors.

### *Absorption Factor*

In absorption operations the inlet vapor feed rate  $V_b$  is specified and the design variable is usually the inlet solvent rate  $L_a$ . To select it, a helpful factor to consider is the *absorption factor*  $A$ , defined as the ratio of the slope of the operating line to the slope of the equilibrium line:

$$A = \frac{L/V}{m} = \frac{y_b - y_a}{y_b^* - y_a^*} \quad (10)$$

Where  $m$  is the slope of the equilibrium line. When the equilibrium line is not straight an average slope may be used, but when the liquid and vapor rates vary significantly from one end of the cascade to the other one must decide which end to use, either  $L_a/V_a$  or  $L_b/V_b$ .

In terms of the compositions in Eq. (10) the inlet composition  $y_b$  is a specification,  $y_a$  is either specified or can be calculated from the specification of the required solute recovery, and  $y_a^*$  is determined from equilibrium with the solvent inlet composition  $x_a$  which is also a specification. Thus  $y_b^*$  can be determined from Eq. (10) as a function of the absorption factor and, by the equilibrium relationship,  $x_b$  can be determined. The solvent rates  $L_a$  and  $L_b$  can then be determined from the over-all balances.

What is a reasonable value of the absorption factor? To answer we can look at the  $x$ - $y$  diagram of Fig. 10. It is evident that an absorption factor of 1.0 (parallel equilibrium and operating lines) results in an unreasonable large number of equilibrium stages, while a value of 10 results in a very dilute extract (low value of  $x_b$ ) with little reduction in the number of stages. The near-optimum value of the absorption factor has been found to be:

$$A = 1.2$$

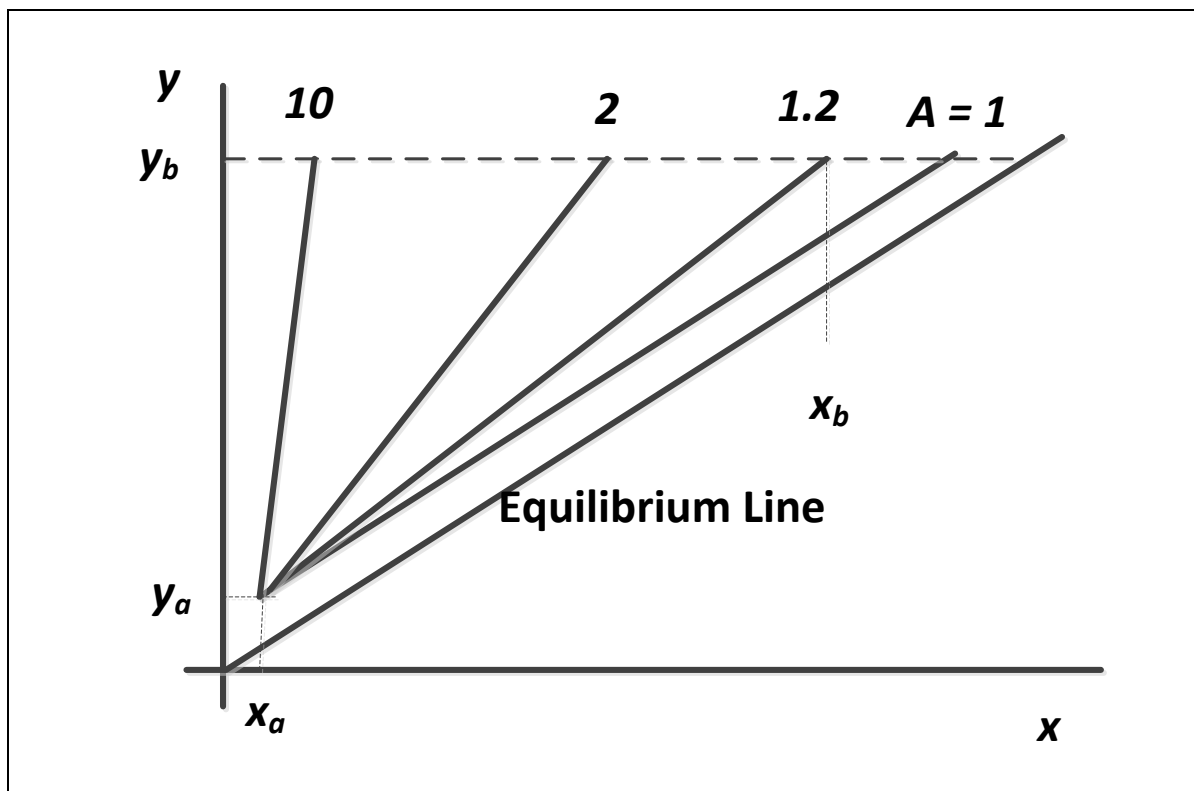


Figure 10. Effect of Absorption Factor on Operating Line

### *Stripping Factor.*

In stripping operations the liquid or heavy phase feed rate  $L_a$  is a specification and the vapor or light phase inlet rate  $V_b$  is the design variable. Other specifications are the inlet compositions  $x_a$  and  $y_b$  and either the liquid outlet composition  $x_b$  or the fraction of solute recovered from which the liquid outlet composition can be determined. The stripping factor is useful in determining a



reasonable vapor inlet rate. It is defined as the ratio of the slope of the equilibrium line to the slope of the operating line:

$$S = \frac{m}{L/V} = \frac{x_a - x_b}{x_a^* - x_b^*} \quad (11)$$

As with the absorption factor, if the equilibrium line is not straight an average slope must be determined for  $m$ , and, if the liquid and vapor rates vary significantly from one end of the cascade to the other, the designer must decide at which end to evaluate the flows, either  $L_b/V_b$  or  $L_a/V_a$ . In terms of the compositions  $x_a$ ,  $x_b$  and  $x_b^*$  can be determined from the specifications,  $x_a^*$  can be determined from Eq. (11) for a value of the stripping factor, then  $y_a$  can be determined from the equilibrium relationship and  $V_a$  and  $V_b$  can be calculated from the over-all balances.

As with the absorption factor a near-optimum value of the stripping factor is 1.2, as illustrated in Fig. 11.

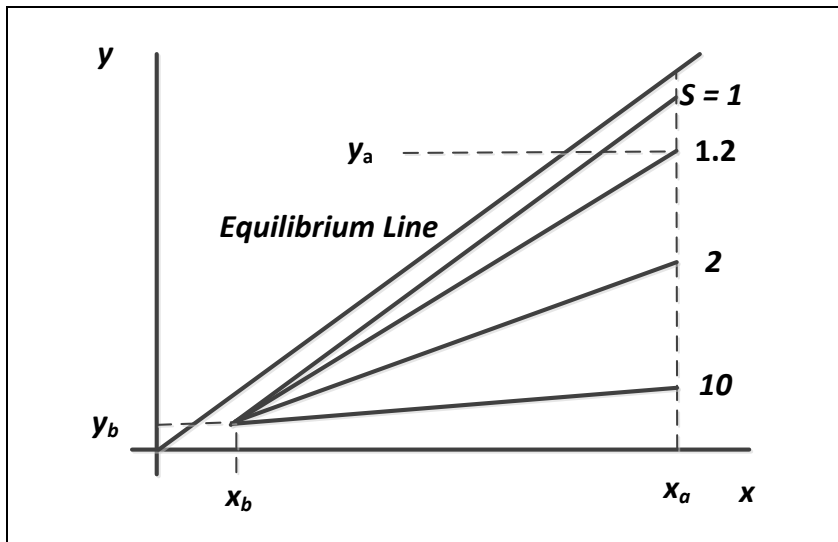


Figure 11. Effect of Stripping Factor on Operating Line

Table 1 shows the effect of the absorption factor on the number of required equilibrium stages. For this problem  $x_a = 0$  (inlet solvent is pure) and the fraction recovery of solute is also varied in the table.

**Table 1. Effect of Absorption Factor on Number of Stages**

Recovery $A = L/mV$	95.0%	98.0%	99.0%	99.5%
	Number of Equilibrium Stages			
1.0	19	49	99	199
1.2	8	12	16	19
2.0	3	5	6	7
10.0	1	2	2	2

Although the number of required stages is low for absorption factors of 2.0 and 10.0, these values are not optimal because they require excessive amount of solvent and produce a very dilute extract that is expensive to process. The near-optimum absorption factor of 1.2 is highlighted.

Having introduced the general design of countercurrent stage operations we will now illustrate it with two applications, gas absorption and stripping. Both of these operations have already been introduced with regard to the design of packed towers. Later we will apply these principles to the design of distillation, leaching and liquid-liquid extraction. It is important to understand that any of these operations can be carried out in packed towers and in stage operations.

## 7 Design of Gas Absorption and Stripping Tray Towers

*Gas absorption* consists of removing a gaseous component or *solute* from a mixture with an inert gas by contacting it with a liquid solvent that absorbs the solute and not the inert gas. *Stripping* consists of removing a solute from a liquid solution by contacting it with an inert gas that strips the solute from the liquid. Therefore each of these operations consists of three components,

- a *solute* that is transferred from one phase to the other,
- a *liquid solvent* that forms the liquid solution with the solute, and
- an *inert gas* that forms a gaseous mixture with the solute.

Standard assumptions for the design of both absorption and stripping operations are,

- the liquid solvent does not vaporize and
- the inert gas does not dissolve into the liquid stream.

Therefore, each phase consists of only two components, the gas phase has only solute and inert gas, and the liquid phase has only solute and solvent. This means that we need only one the composition of the solute to describe each phase, as the composition of the other component in the phase is just the balance which is not solute.

The flows in absorption and stripping must usually be in **molar units**, that is, lbmole/hr or kmole/hr, and therefore the compositions must be in **mol fractions**. This is because the equilibrium relationships usually relate the mole fractions of the solute in the liquid and vapor phases.

### 7.1 Molar Balances.

With three components we can write three independent molar balances. Depending on the problem specifications, the convenient balances to write can be the three component balances or a total mole balance and two component balances, one of which must always be the solute balance.

### 7.2 Equilibrium.

The equilibrium in absorption and stripping consists of the solubility of the solute in the solvent. Experimental data must be obtained and correlated to do the calculations. For some systems it can be assumed that the solute obeys either Raoul's law or Henry's law, which case the equilibrium line is linear if the tower operates at uniform temperature and pressure.

Raoult's Law:

$$y^* = \frac{P_A^o(T)}{P} x \quad ( 12 )$$

Henry's Law:

$$y^* = \frac{H_A(T)}{P} x \quad ( 13 )$$

Where  $y^*$  = vapor mole fraction in equilibrium with the liquid

$x$  = liquid mole fraction

$P$  = total absolute pressure

$P_A^o(T)$  = vapor pressure of solute at tower temperature  $T$

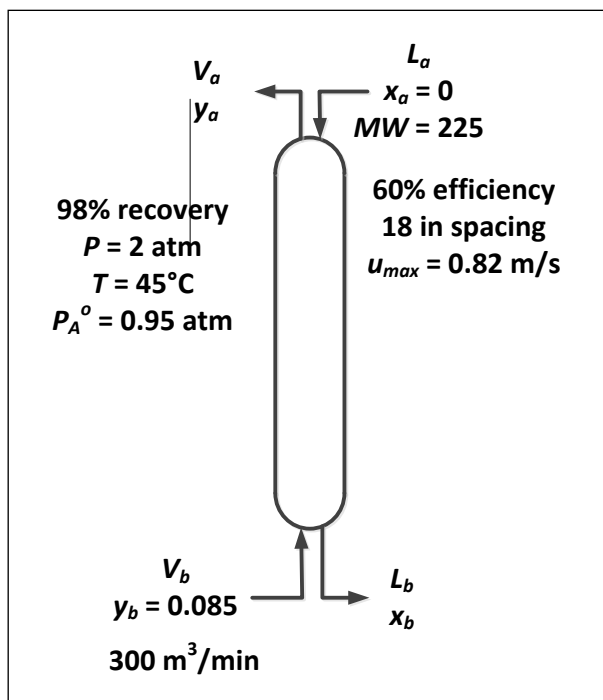
$H_A(T)$  = Henry's law constant of solute at tower temperature  $T$

Henry's law data is applicable when the critical temperature of the solute is lower than the tower temperature.

### Example 1. Design of a Gas Absorption Tray Tower

Design a tray tower to remove 98% of the pollutant in a waste gas stream entering at the rate of 300 m<sup>3</sup>/min at STP (1 atm, 0°C) and a composition of 8.5 mole% pollutant. The tower operates at 2 atm and 45°C. At this temperature the vapor pressure of the pollutant is 0.95 atm. A solvent with negligible vapor pressure and a molecular weight of 225 is used. Determine the required number of actual trays for 60% over-all efficiency and a reasonable solvent rate, the height of the tower for a tray spacing of 18 inches, and the diameter of the tower for a maximum superficial vapor velocity of 0.82 m/s. Report also the inlet flow of solvent in kg/min.

**Solution.** The first thing to do when solving a design problem is to draw a schematic of the equipment, label all streams in and out, and enter the specification from the problem on the diagram.



As the composition of the solvent in is not specified we assume pure solvent,  $x_a = 0$ .

At STP of 1 atm and 0°C, the gas volume is 22.4 m<sup>3</sup>/kmole. So the gas feed rate is:

$$V_b = (300 \text{ m}^3/\text{min}) / (22.4 \text{ m}^3/\text{kmole}) \times$$

$$(60 \text{ min/hr}) = 804 \text{ kmole/hr}$$

Assume Raoult's law applies:

$$y^* = (0.95 \text{ atm}) / (2 \text{ atm}) x = 0.475x$$

$$\text{So, } m = 0.475.$$

$$\text{Solute in exit gas: } V_a y_a = (1 - 0.98)(804 \text{ kmole/hr})(0.085) = 1.37 \text{ kmole/hr}$$

$$\text{Inert gas in exit gas: } V_a(1 - y_a) = V_b(1 - y_b) = (804 \text{ kmole/hr})(1 - 0.085) =$$

$$736 \text{ kmole/hr}$$

$$\text{Exit gas: } V_a = 1.37 + 736 = 737 \text{ kmole/hr}$$

$$\text{Exit gas mole fraction: } y_a = 1.37 / 737 = 0.00185$$

Now we must decide on a reasonable solvent rate. Let's use an absorption factor of  $A = 1.2$ . As the vapor and liquid rates vary from the top to the bottom of the column, so does the absorption factor. Let us assume the absorption factor is 1.2 at the top of the column:

$$A = 1.2 = \frac{L_a}{mV_a} = \frac{L_a}{(0.475)(737)} \quad L_a = (1.2)(0.475)(737) = 420 \text{ kmole/hr}$$

$$\text{Total balance: } L_b = L_a + V_b - V_a = 420 + 804 - 737 = 487 \text{ kmole/hr}$$

$$\text{Solute balance: } x_b = \frac{V_b y_b + L_a(0) - V_a y_a}{L_b} = \frac{(804)(0.085) + 0 - (737)(0.00185)}{487} = 0.138$$

With these numbers, the absorption factor at the bottom of the column is:

$$A = L_b / mV_b = 487 / (0.475)(804) = 1.28$$

The following is a summary of the flows and compositions of the inlet and outlet streams already calculated from the over-all balances:

$$V_b = 804 \text{ kmole/hr} \quad y_b = 0.085 \quad V_a = 737 \text{ kmole/hr} \quad y_a = 0.00185$$

$$L_b = 487 \text{ kmole/hr} \quad x_b = 0.138 \quad L_a = 420 \text{ kmole/hr} \quad x_a = 0$$

The next step is to plot the equilibrium and operating lines on the  $x$ - $y$  diagram to determine the number of equilibrium stages required. The equilibrium line is straight so that only two points are required to plot it, say  $y = 0$  at  $x = 0$ , and  $y = 0.475(0.10) = 0.0475$  at  $x = 0.10$ .

The operating line is curved because the vapor and liquid flows vary throughout the column, so more than two points are required to plot it. The line goes through the points  $y_b = 0.085$  at  $x_b = 0.138$ , and  $y_a = 0.00185$  at  $x_a = 0$ . If the operating line is approximately straight it can be easily drawn with a ruler through these two points. To check if the line is approximately straight we can calculate its slope  $L/V$  at both ends of the column and, if it varies by less than about 5%, it is close enough to straight. In this example:

$$L_a/V_a = 420/737 = 0.57 \qquad L_b/V_b = 487/804 = 0.60$$

These two are close enough for the line to be plotted with a ruler through the two end points, but to illustrate the procedure we will calculate a third point. Usually one more point in the middle of this range is sufficient to sketch the line. We use the operating line to calculate.

$$\text{Let } x = 0.070 \qquad L = L_a(1 - x_a)/(1 - x) = 420(1 - 0)/(1 - 0.070) = 452 \text{ kmole/hr}$$

$$V = V_a + L - L_a = 737 + 452 - 420 = 769 \text{ kmole/hr}$$

$$y = (V_a y_a + Lx - L_a x_a)/V = (737 \times 0.00185 + 452 \times 0.070 - 420 \times 0)/769 = 0.043$$

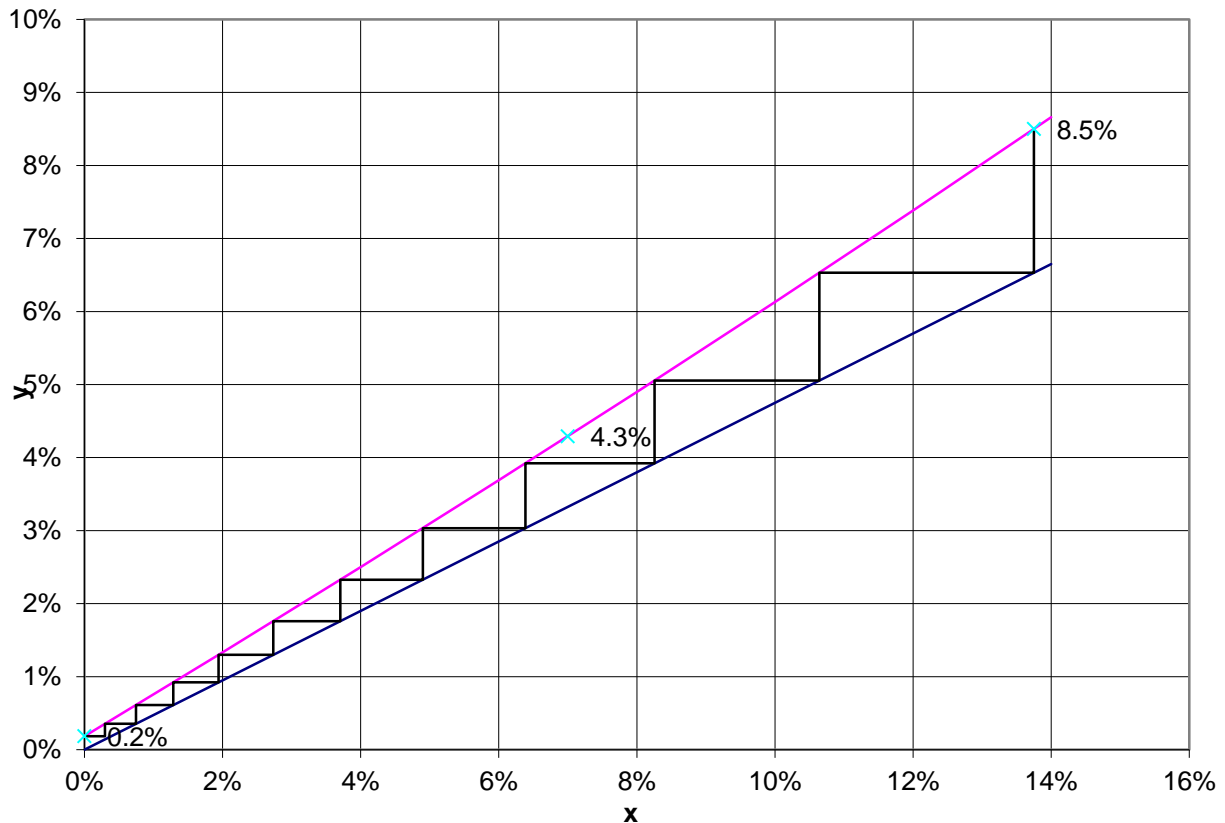
So the third point on the operating line is  $y = 0.043$  at  $x = 0.070$ . The  $x$ - $y$  diagram shows the operating and equilibrium lines and the stepping of the stages. The separation requires 10.8 equilibrium stages.

The rest of the design is now calculated. For 60% over-all efficiency and 18-in tray spacing:

$$\text{Number of actual stages:} \qquad 10.8/0.60 = 18 \quad \text{actual trays}$$

Tower height:

$$18(18 \text{ inch})(1 \text{ ft}/12 \text{ inch}) = \underline{27 \text{ ft } (8.2 \text{ m})}$$



As the gas velocity in the tower cannot be greater than the maximum, the diameter must be determined at the bottom where the gas flow is the highest.

$$\text{Volumetric flow: } \frac{804 \text{ kmole}}{\text{hr}} \frac{22.4 \text{ m}^3 \text{ STP}}{\text{kmole}} \frac{(273+45) \text{ K}}{273 \text{ K}} \frac{1 \text{ atm}}{2 \text{ atm}} \frac{1 \text{ hr}}{3600 \text{ s}} = 2.91 \frac{\text{m}^3}{\text{s}}$$

$$\text{Tower area: } \frac{\pi D^2}{4} = \frac{2.91 \text{ m}^3}{\text{s}} \frac{\text{s}}{0.82 \text{ m}} = 3.55 \text{ m}^2$$

$$\text{Tower diameter: } D = \sqrt{\frac{4(3.55 \text{ m}^2)}{\pi}} = 2.13 \text{ m } \underline{(7.0 \text{ ft})}$$

$$\text{Inlet solvent flow: } (420 \text{ kmole/hr})(\text{hr}/60 \text{ min})(225 \text{ kg/kmole}) = \underline{1575 \text{ kg/min}}$$



Before we leave this example, let us check what results we would have gotten had we estimated the number of stages using the Kremser equation.

$$y_b^* = 0.475x_b = 0.475(0.138) = 0.0655 \quad y_a^* = 0.475(0) = 0$$

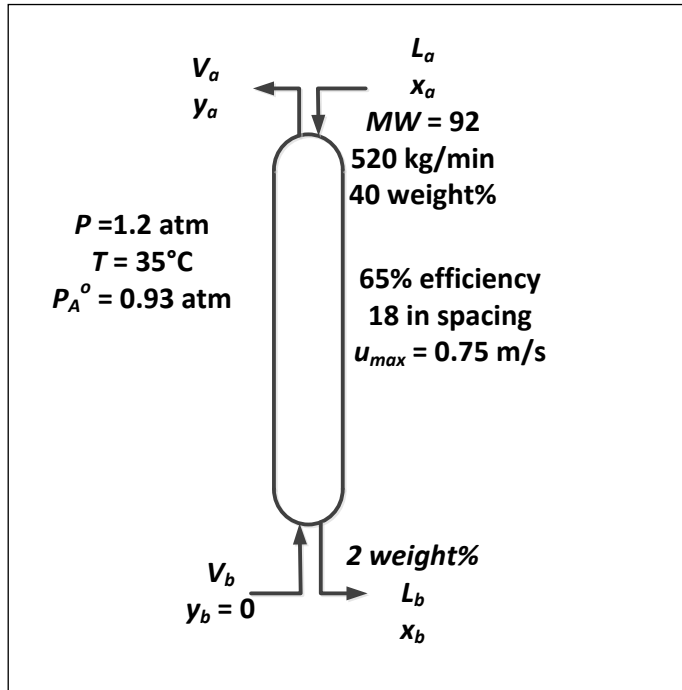
$$N = \frac{\ln\left(\frac{0.085-0.0655}{0.00185-0}\right)}{\ln\left(\frac{0.085-0.00185}{0.0655-0}\right)} = 9.9 \text{ stages}$$

Compare with 10.8 stages with the McCabe-Thiele method. The reason the Kremser equation predicts slightly fewer stages is that, as can be seen in the  $x$ - $y$  diagram, in this case the operating line curves closer to the equilibrium line. As the Kremser equation is based on a straight operating line between the two end points, such a line would be farther from the equilibrium line and require fewer stages as each stage would cause a larger change in mole fraction than the actual curved line.

### Example 2. Design of a Tray Stripping Tower

A tray tower is to remove a contaminant from process water by contacting it with air. The feed consists of 520 kg/min of water containing 40 weight% of the contaminant. The water leaving the tower must contain no more than 2 weight% of the contaminant. The contaminant has a molecular weight of 92. Design the tower using a reasonable rate of air. Report the number of actual trays required for an over-all efficiency of 65%, the tower height for an 18 inch tray spacing, the tower diameter for a maximum superficial vapor velocity of 0.75 m/s. Report also the air rate in m<sup>3</sup>/min at STP (1 atm and 0°C). The tower operates at 1.2 atm and 35°C. At this temperature the vapor pressure of the contaminant is 0.93 atm.

**Solution.** The first step is to draw a schematic of the tower labeling all the inlet and outlet streams and entering all the problem data. We assume the water does not vaporize, the air does not dissolve in the water, and the inlet air is pure. We also assume Raoult's law applies to the contaminant.



Feed composition (mol. wt. of water is 18):

$$x_a = \frac{\frac{40\%}{92}}{\frac{40\%}{92} + \frac{60\%}{18}} = 0.115$$

Average molecular weight of feed:

$$MW_a = \frac{0.115(92)}{0.115(92) + 0.885(18)} = 26.5$$

Molar rate in:

$$L_a = \frac{520 \text{ kg}}{\text{min}} \frac{60 \text{ min}}{\text{hr}} \frac{\text{kmole}}{26.5 \text{ kg}} = 1176 \frac{\text{kmole}}{\text{hr}}$$

Composition of exit water: 
$$x_b = \frac{\frac{2\%}{92}}{\frac{2\%}{92} + \frac{98\%}{18}} = 0.0040$$

We have four unknowns,  $L_b$ ,  $V_b$ ,  $V_a$  and  $y_a$ , and three mass balances for three components. The problem is underdefined, so we must choose a reasonable air inlet rate  $V_b$ .

Water balance: 
$$L_b = L_a \frac{1-x_a}{1-x_b} = \frac{1176 \text{ kmole}}{\text{hr}} \frac{1-0.115}{1-0.004} = 1044 \frac{\text{kmole}}{\text{hr}}$$

To select a reasonable air rate use a stripping factor of 1.2 at the bottom:

$$S = 1.2 = \frac{mV_b}{L_b}$$

Equilibrium (Raoult's law):  $y^* = mx$        $m = \frac{0.93atm}{1.2atm} = 0.775$

$$V_b = SL_b/m = 1.2(1044)/0.775 = 1617 \text{ kmole/hr}$$

Total balance:  $V_a = L_a + V_b - L_b = 1176 + 1617 - 1044 = 1749 \text{ kmole/hr}$

Solute balance:  $V_a y_a = L_a x_a + V_b y_b - L_b x_b = 1176(0.115) + (0) - 1044(0.004)$   
 $= 131 \text{ kmole/hr}$

Vapor rate out:  $y_a = V_a y_a / V_a = 131/1749 = 0.0749$

Summary of the flows and compositions of the inlet and outlet streams:

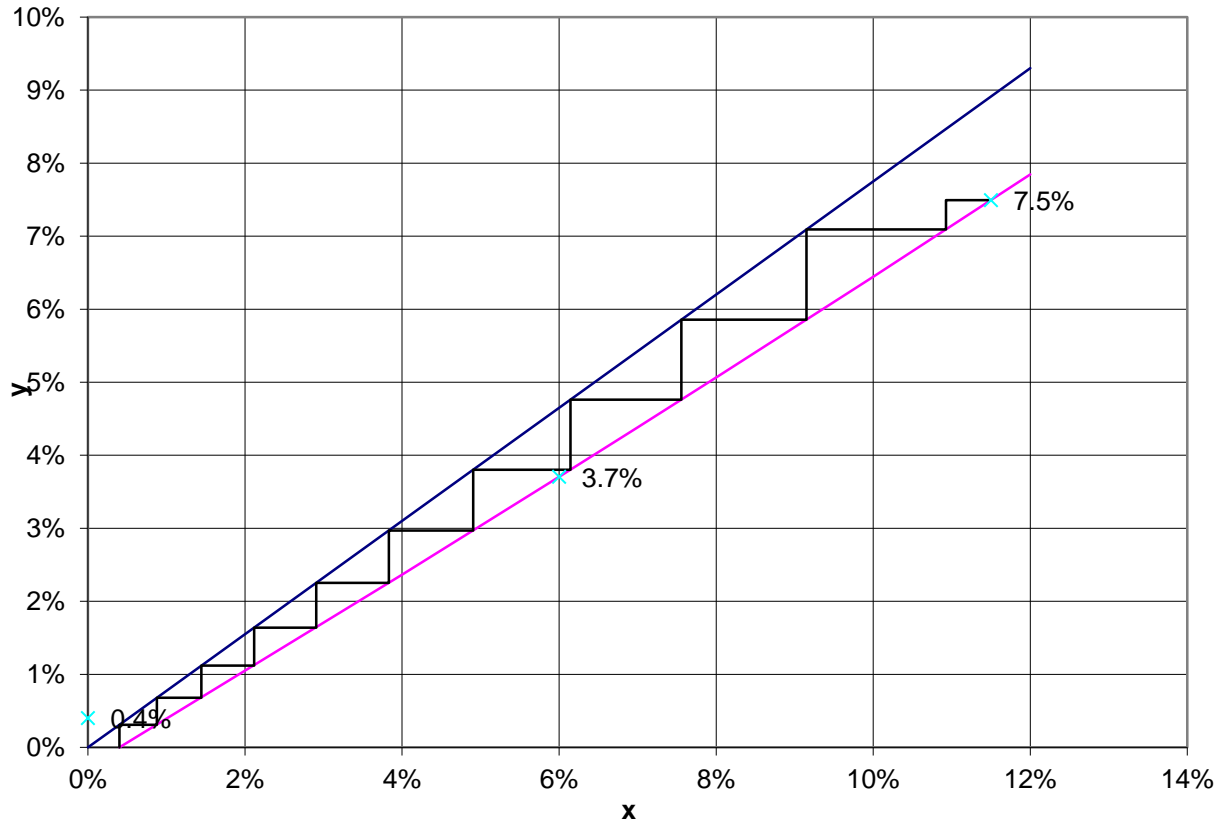
$$L_a = 1176 \text{ kmole/hr} \quad x_a = 0.115 \quad L_b = 1044 \text{ kmole/hr} \quad x_b = 0.0040$$

$$V_a = 1749 \text{ kmole/hr} \quad y_a = 0.0749 \quad V_b = 1617 \text{ kmole/hr} \quad y_b = 0$$

The equilibrium line is straight with a slope of 0.775. Let us check if the operating line is approximately straight:

$$L_a/V_a = 1176/1749 = 0.67 \quad L_b/V_b = 1044/1617 = 0.65$$

These two slopes are close enough to permit drawing the operating line with a ruler through the two end points,  $y_b = 0$  at  $x_b = 0.0040$ , and  $y_a = 0.0749$  at  $x_a = 0.115$ . A third intermediate point is not needed in this case.



The required number of equilibrium stages is 10.1. We can now complete the design calculations.

Actual number of stages with 65% efficiency:  $10.1/0.65 = 15.5$  16 actual trays

Tower height, 18-in spacing:  $16(18 \text{ in})(1\text{ft}/12 \text{ in}) = 24.0 \text{ ft}$  7.3 m

As the velocity cannot be greater than 0.75 m/s, the tower must be sized at the top where the vapor flow is the highest,  $V_a = 1749 \text{ kmole/hr}$ .

Volume vapor flow at 35°C, 1.2 atm:  $\frac{1749 \text{ kmole}}{\text{hr}} \frac{22.4 \text{ m}^3}{\text{kmole}} \frac{(273+35)\text{K}}{273\text{K}} \frac{1 \text{ atm}}{1.2 \text{ atm}} \frac{\text{hr}}{3600 \text{ s}} = 10.2 \frac{\text{m}^3}{\text{s}}$

Tower area:  $\frac{\pi D^2}{4} = \frac{10.2 \text{ m}^3}{\text{s}} \frac{\text{s}}{0.75 \text{ m}} = 13.6 \text{ m}^2$

Tower diameter:  $\sqrt{\frac{4(13.6 \text{ m}^2)}{\pi}} = 4.2 \text{ m}$  13.7 ft

$$\begin{aligned}\text{Inlet air flow, } V_b &= (1617 \text{ kmole/hr})(22.4 \text{ m}^3/\text{kmole})(\text{hr}/60 \text{ min}) \\ &= \underline{604 \text{ m}^3/\text{min at STP (1 atm, 0}^\circ\text{C)}}\end{aligned}$$

Let's check the number of equilibrium stages from the Kremser equation:

$$x_b^* = 0/0.775 = 0 \quad x_a^* = 0.0749/0.775 = 0.0966 \quad x_b = 0.0040 \quad x_a = 0.115$$

$$N = \frac{\ln\left(\frac{0.115-0.0966}{0.0040-0}\right)}{\ln\left(\frac{0.115-0.0040}{0.0966-0}\right)} = 10.9$$

Compare with 10.1 stages by the McCabe-Thiele procedure. The close agreement is because the equilibrium line is straight and the operating line is approximately straight, as is evident from the  $x$ - $y$  diagram.

### 7.3 Stage by Stage Calculation

The McCabe-Thiele graphical solution is fast and easy, but, being a graphical procedure, is not very accurate. The same procedure can be carried out by doing the calculations stage by stage, but this is very tedious to do when the required number of stages is high, except if a spreadsheet or workbook is used to do the calculations. As with any workbook calculation it is a requirement to check out the formulas by first doing a sample calculation, as follows.

**Sample calculation.** Start with the bottom stage, Stage 1:

$$\text{Out of Stage 1: } L_b = L_1 = 1044 \text{ kmole/hr} \quad x_b = x_1 = 0.40\%$$

$$\text{By equilibrium: } y_1 = 0.775(0.40\%) = 0.31\%$$

$$\text{Air balance on Stage 1: } V_1 = V_b \frac{1-y_b}{1-y_1} = 1617 \frac{1-0}{1-0.0031} = 1622 \text{ kmole/hr}$$

Total balance, Stage 1:  $L_2 = V_1 + L_b - V_b = 1622 + 1044 - 1617 = 1049$  kmole/hr

$$\text{Solute balance, Stage 1: } x_2 = \frac{(V_1 y_1 + L_b x_b - V_b y_b)}{L_2} = \frac{1622(0.31\%) + 1044(0.40\%) - 1617(0)}{1049} = 0.88\%$$

These calculations are repeated for each stage until  $y_N$  is greater than  $y_a$ . Then it is replaced with the value of  $y_a$ , in this case 7.49%. The following workbook presents the rest of the calculations:

Stage	x	y	kmole/hr	kmole/hr
			V	L
b		0.00%	1617	
1	0.40%	0.31%	1622	1044
2	0.88%	0.68%	1629	1049
3	1.45%	1.13%	1636	1056
4	2.13%	1.65%	1645	1063
5	2.94%	2.28%	1656	1072
6	3.88%	3.01%	1668	1083
7	4.99%	3.86%	1683	1095
8	6.27%	4.86%	1701	1110
9	7.74%	6.00%	1722	1128
10	9.42%	7.30%	1746	1149
a	11.50%	7.49%	1749	1176

Notice that the sample calculations agree with the calculations in the cells for Stage 1 and for  $x_2$  and  $L_2$ . As these cells have been checked for accuracy the rest of the cells are sure to be correct as they are copied and pasted from them.

The calculation of the fraction of stage for the last stage, Stage N, is done as follows:

$$N = (N - 1) + \frac{y_a - y_{N-1}}{m x_N - y_{N-1}} = 10 + \frac{7.49\% - 7.30\%}{0.775(11.50\%) - 7.30\%} = 10.1 \text{ stages}$$

This is the same result as the graphical solution because those stages were calculated with a workbook similar to this one.

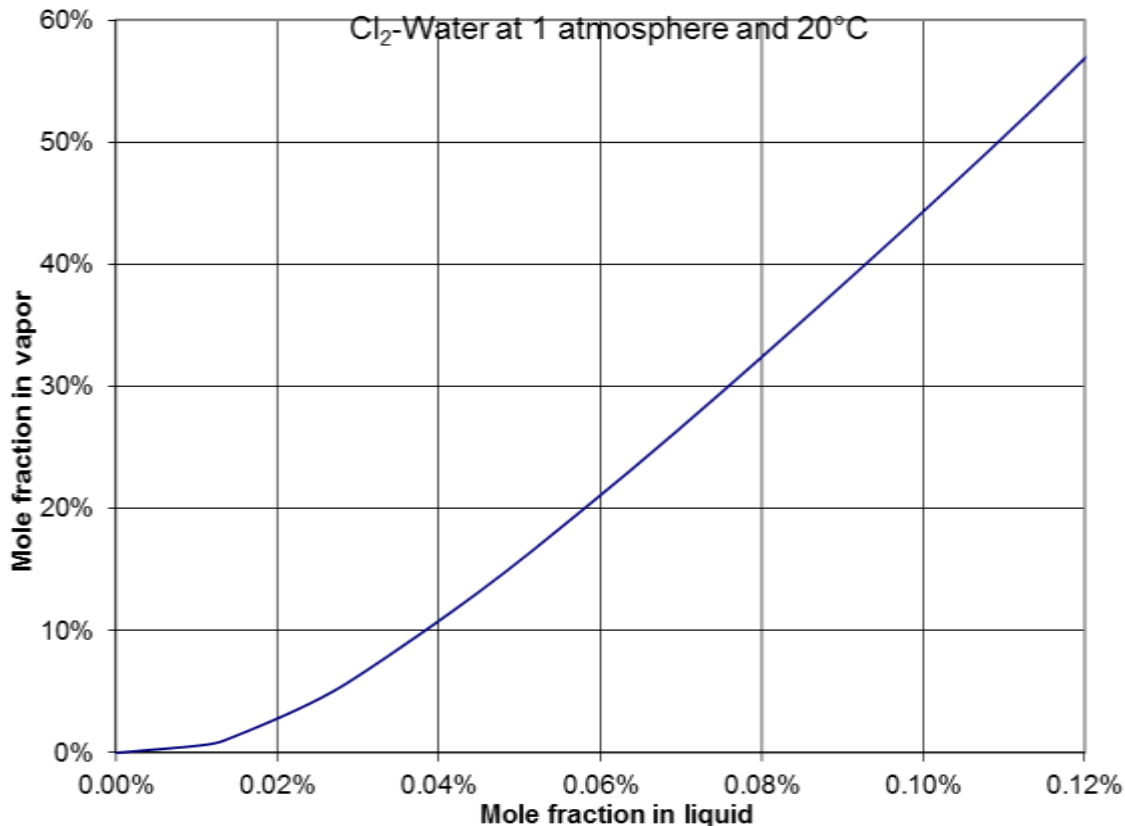
### **Study Questions**

1. Describe what a countercurrent stage separation consists of.
2. What is an *equilibrium stage*?
3. Why is the number of real stages required always higher than the number of equilibrium stages?
4. What is a *design variable*? Why is it necessary to select its value?
5. Which variables are related by the over-all balances on a cascade?
6. How is the number of equilibrium stages determined?
7. Briefly describe the McCabe-Thiele graphical procedure. What is its purpose?
8. Briefly describe how are the equilibrium and operating lines determined.
9. Cite the restrictions for the Kremser equation to be valid. What is its purpose?
10. Describe the *absorption factor* and the *stripping factor*. What are their near-optimum values?
11. Cite the assumptions commonly made for designing absorption and stripping towers.
12. How many over-all mass balances are independent in absorption or stripping tower design?
13. What are the relative positions of the operating and equilibrium lines in absorption operations? What are they for stripping operations? Why must they be that way?
14. When stage by stage calculations are performed in a spreadsheet or workbook, why must sample calculations be presented?

## Problems

1. **Butane Absorption from Air.** (Based on data from McCabe, Smith, and Harriott, 7th ed., Problem 20.7.) A mixture of 12.5 mole% butane and the balance air is fed to a sieve-tray absorber operating at 2 atm and 15°C, at the rate of 500 m<sup>3</sup>/min at STP (1 atm, 0°C). At 15°C the vapor pressure of butane is 1.92 atm. The absorbing liquid is a nonvolatile oil having a molecular weight of 250 and a specific gravity of 0.90. Draw a schematic of the absorber showing all the problem data and, using a reasonable oil rate, design the absorber to recover 98% of the butane. Report the number of actual trays for an over-all efficiency of 70%, the height of the column for a tray spacing of 18 inches, and the column diameter for a maximum superficial vapor velocity of 0.95 m/s. Report also the inlet oil rate in liters/min. State all assumptions.
2. **Design of a Tray Cl<sub>2</sub> absorber.** A tray column is to remove 99% of the Cl<sub>2</sub> in a stream containing 43 mole% Cl<sub>2</sub> and the balance air using pure water as the solvent. The column is to operate at 1 atm and 20°C. The equilibrium relationship at these conditions is given in the attached x-y diagram. The feed rate is 15 m<sup>3</sup>/min at STP (1 atm and 0°C). Draw a schematic of the column showing all the problem data on the schematic and determine the required number of actual stages for an over-all column efficiency of 65%, the tower height for a tray spacing of 18 inches, and the column diameter for a maximum superficial vapor velocity of 1.0 m/s. Report also the feed rate of water in liters/min. State all assumptions.



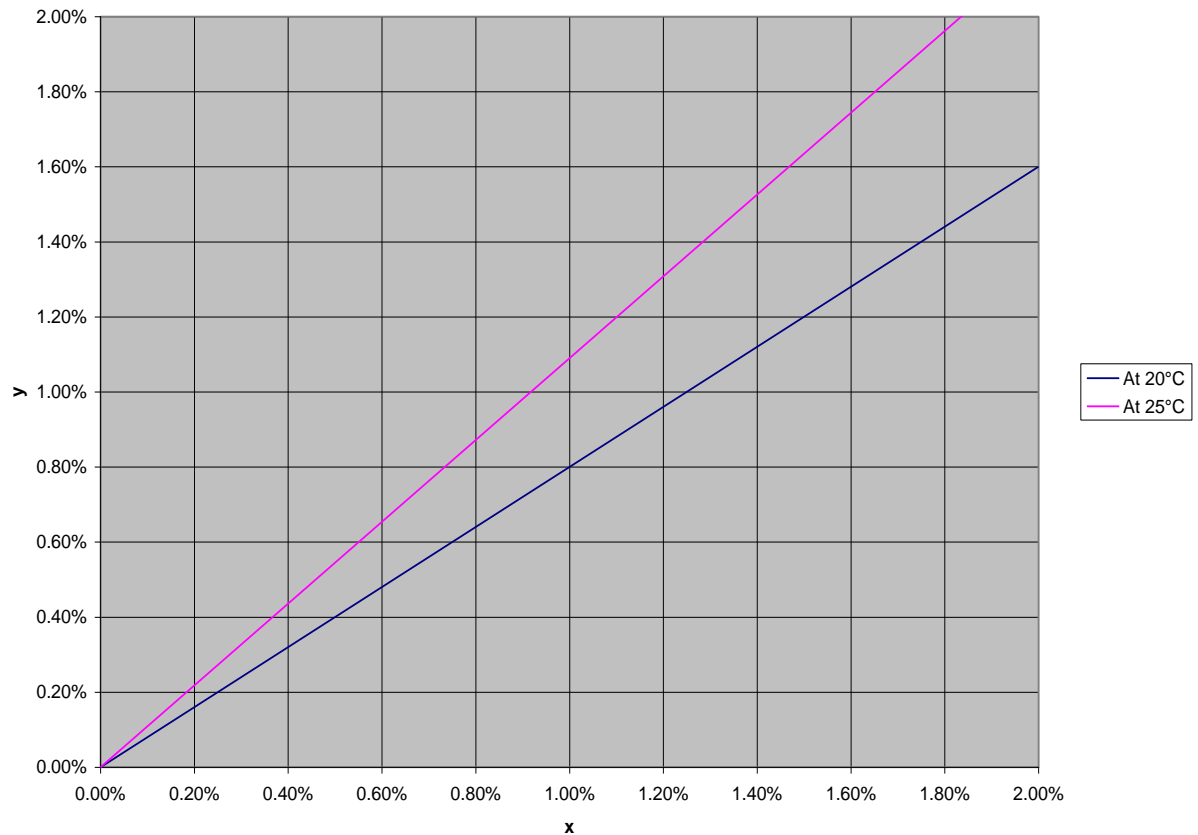


3. **Design of a Chlorine Tray Stripper.** It is desired to remove the chlorine from waste water by stripping it with air in a tray tower. The feed, at the rate of 600 liters/min contains 0.45 weight% chlorine and the balance water. You may assume the specific gravity is the same as water. The water exiting the tower must contain no more than 0.1 weight% chlorine. Draw a schematic of the tower showing all the problem data and design it using a reasonable air rate. Report the required number of actual trays for an overall efficiency of 70%, the tower height for 18-inch tray spacing, and the tower diameter for a maximum superficial vapor velocity of 0.9 m/s. Report also the air inlet rate in m<sup>3</sup>/min at STP (1 atm, 0°C). The tower runs at 1 atm and 20°C so the equilibrium data for Problem 2 applies.(Based
4. **Design of a Benzene Tray Absorber.** (SACHE Problem 77). Benzene is used in a process as a solvent for a solid product, and it is dried from the solid at the end of the process. Since benzene is quite flammable (its LFL is 1.3 percent) and toxic (its permissible exposure limit is 10 parts per million),

nitrogen is recycled as a carrier gas during drying. Neither the nitrogen nor the benzene is ever to be released from the process. In order to recycle both the benzene (as a liquid solvent) and the nitrogen (as a carrier in the drying process), the benzene in the nitrogen is removed in a tray absorber. The benzene entering the absorber is at a concentration of 7.4 mole percent in nitrogen. It must be reduced to a concentration of 0.4 mole percent in nitrogen, after which the nitrogen stream will be heated and recycled to dry the product. The benzene will be absorbed in an oil having a molecular weight of 200. Raoult's law can be assumed to apply, and the absorber is designed to operate at 50°C (because the nitrogen-benzene stream entering is hot) and 1.0 atm. The vapor pressure of the oil is negligible, and the nitrogen can be assumed to be insoluble in the oil. Draw a schematic of the tower showing all the problem data on the sketch and determine the mole fraction of benzene in the liquid leaving the absorption tower and the number of actual trays required for the process at an over-all tower efficiency of 60% and the tower height for a 12-in tray spacing. Use a reasonable solvent rate. The Antoine constants for the vapor pressure of benzene in mm Hg are:  $A = 15.9008$ ,  $B = 2788.51 \text{ K}$ ,  $C = -52.36 \text{ K}$  (from Reid, Prausnitz and Sherwood, *The Properties of Gases and Liquids*, 3<sup>rd</sup> ed., McGraw Hill, 1977, Appendix A).

5. **Design of a tray ammonia absorber.** A tray column is to remove 98% of the ammonia in a gas stream containing 1.7 mole% ammonia and the balance air using water as the solvent. The tower operates at 1 atm. Draw a schematic of the column showing all the problem data and design the column using a reasonable solvent rate. Report the number of actual trays required for an over-all efficiency of 70%, the height of the tower for a tray spacing of 24 inches. Determine also the solvent rate for an inlet gas rate of 300 m<sup>3</sup>/min at STP (0°C and 1 atm) and the tower diameter for a maximum superficial gas velocity of 1.1 m/s. State all assumptions.

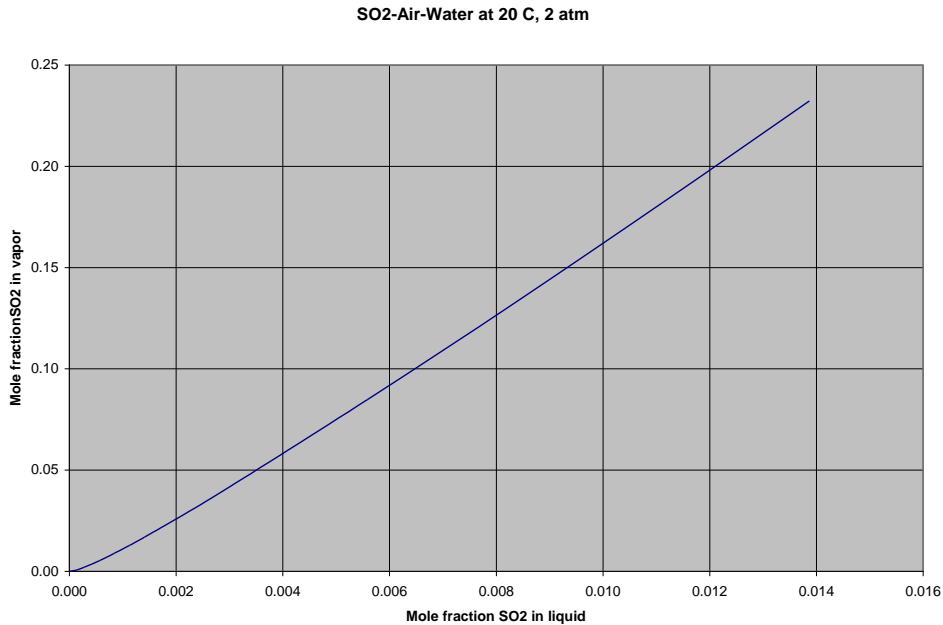
The following are the equilibrium lines at the two temperatures. Select the temperature that requires less solvent resulting in a more concentrated extract.



6. **Design of a Tray Ammonia Stripper.** A solution containing 1.8 weight% ammonia in water is fed to a tray tower to remove 96% of the ammonia by stripping it with air. The tower operates at 1 atm. Draw a schematic of the tower showing all the problem data and, using a reasonable air rate, design the tower. Report the required number of actual stages for an over-all efficiency of 60% and the tower height for a tray spacing of 24 inches. If the feed rate is 120 kg/min, determine the tower diameter for a maximum superficial vapor velocity of 0.90 m/s, and the rate of air in m<sup>3</sup>/min at STP (1 atm, 0°C). The equilibrium data

at two different temperatures is given with Problem 5. Decide at which temperature to run the tower to use the smaller amount of air and produce a more concentrated exit gas stream. State all assumptions.

7. **Design of a Tray Benzene Absorption Tower.** A countercurrent tray absorption tower is fed at the bottom with  $55 \text{ m}^3/\text{min}$  at STP (1 atm,  $0^\circ\text{C}$ ) of a gas containing 6 mole% benzene and the balance air. It is desired to recover 97% of the benzene in the feed by countercurrent contact with non-volatile oil which enters at the top of the tower. The column operates at 2 atmospheres pressure and  $27^\circ\text{C}$ . At  $27^\circ\text{C}$  the vapor pressure of benzene is 103.6 mmHg. Draw a schematic of the tower and show all the problem data on the schematic and design the tower using a reasonable solvent rate. Report the required number of actual trays for an over-all efficiency of 60%, the height of the tower for a 24-inch tray spacing, and the tower diameter for a maximum superficial vapor velocity of  $0.85 \text{ m/s}$ . Report also the solvent rate in  $\text{kmole/hr}$ . State all assumptions.
8. **Design of a Tray  $\text{SO}_2$  Absorber.** A tray column is to remove 98% of the  $\text{SO}_2$  in a stream containing 20 mole%  $\text{SO}_2$  and the balance air using pure water as the solvent. The column is to operate at 2 atm and  $20^\circ\text{C}$ . The equilibrium relationship at these conditions is given in the attached x-y diagram. The feed rate is  $120 \text{ m}^3/\text{min}$  at STP (1 atm,  $0^\circ\text{C}$ ). Draw the schematic of the absorber showing all the problem data and design it using a reasonable solvent rate. Report the number of actual trays for an over-all efficiency of 70%, the tower height for a 18-inch tray spacing, and the tower diameter for a maximum superficial vapor velocity of  $0.75 \text{ m/s}$ . Report also the inlet solvent rate in  $\text{liters/min}$ . State all assumptions.



9. **Design of a Tray SO<sub>2</sub> Stripper.** A tray column is to remove 98% of the SO<sub>2</sub> in a liquid stream containing 5.41 weight% SO<sub>2</sub> and the balance water by bubbling air through it. The tower is to operate at 2 atm and 20°C. The equilibrium relationship at these conditions is given in the  $x$ - $y$  diagram for Problem 8. Draw the schematic for the stripper showing all the problem data and design it using a reasonable air rate. Report the number of actual trays required for an over-all efficiency of 60% and the height of the tower for a 24-inch tray spacing. For a feed rate is 570 liters/min determine the tower diameter for a maximum superficial vapor velocity of 0.85 m/s. Report also the flow of air in m<sup>3</sup>/min at STP (1 atm, 0°C). Assume the specific gravity of the feed is 1.08. State all assumptions.

10. ***Design of a Tray Benzene Absorption Tower.*** A countercurrent tray absorption column is fed at the bottom with an air stream containing 1.5 mole% benzene and the balance air. It is desired to recover 99% of the benzene in the feed by countercurrent contact with a non-volatile oil which enters at the top of the column containing 0.2 mole% benzene. The column operates at 27°C. At this temperature the vapor pressure of benzene is 103.6 mmHg. Draw a schematic showing all the problem data on the schematic and design the column using a reasonable pressure and solvent rate. Report the required number of real trays for an over-all column efficiency of 75% and the column height for a tray spacing of 18 inches. Caution: as the inlet solvent is not pure, if the column pressure is too low the equilibrium and operating lines may cross. State all assumptions.